P425/1
PURE MATHEMATICS
Paper 1
Jul. / Aug. 2019
3 hours

''Together for Mathematics''

SECONDARY MATHEMATICS TEACHERS' ASSOCIATION

SMATA JOINT MOCK EXAMINATIONS 2019

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** the ***eight*** questions in Section **A** and **five** questions from Section **B**.

Any additional question(s) answered will **not** be marked.

All working **must** be shown clearly.

Begin each answer on a **fresh** sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)

*Answer **all** questions in this Section.*

1. Given that $a=\log_5 35$ and $b=\log_9 35$, prove that $\frac{1}{2b}(2ab-2b+a)=\log_5 21$.

(05 marks)

2. Solve the equation $(2\cot 2x-1)^2=3(\cos ec^2 2x-2)$ for $0^o \le x \le 200^o$.

(05 marks)

3. Differentiate $y=\sec^{-1} e^{2x}$. ***(05 marks)***

4. Given that α and β are the roots of the equation $2x^2-3x+4=0$, form an equation whose roots are $\frac{1}{5\alpha+\beta}$ and $\frac{1}{5\beta+\alpha}$. ***(05 marks)***

5. Show that the points $A(1,\ 2,\ 3)$, $B(3,\ 3,\ 4)$ and $C(4,\ 6,\ 6)$ are vertices of a triangle ABC. ***(05 marks)***

6. Evaluate: $\int_0^1 x\sqrt{(4-3x^2)}\,dx$ ***(05marks)***

7. Find the locus of a point which moves such that its distance from the point $A(2,\ 0)$ is half its distance from the line $x=8$. ***(05marks)***

8. Solve the differential equation $\frac{xy}{x+1}=\frac{dy}{dx}$, given that $y(0)=1$

(05marks)

SECTION B: (60 MARKS)

Answer only ***five*** questions in this section.

9 (a) Find the possible values k given that the point $(4,\ k)$ is the same distance from $9x+8y+1=0$ as $(2,\ 5)$is from $12x-y+2=0$. ***(05 marks)***

(b) Show that the parametric equations $x=5+\frac{\sqrt{3}}{2}\cos\theta$, $y=-3+\frac{\sqrt{3}}{2}\sin\theta$ represent a circle and find the centre and radius. ***(07 marks)***

10 a) Given that $y = In\left(x + \sqrt{x^2 + a^2}\right)$, where a is a constant, prove that $\frac{dy}{dx} = \frac{1}{\sqrt{x^2 + a^2}}$ and hence evaluate $\int_0^4 \frac{dx}{\sqrt{x^2 + 9}}$. **(06 marks)**

b) Find Maclaurin's expansion of $y = In\frac{(2-x)^2}{(1+x)^2}$, showing the first three non zero terms. **(06 marks)**

11 a) Solve: $\left(3x^2 + 2x\right)^2 + 8 = 27x^2 + 18x$. **(06 marks)**

b) Solve the equation: $\sqrt{4x+13} - \sqrt{x+1} = \sqrt{12-x}$. **(06 marks)**

12 a) If the position vectors of points A and B are $2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$ and $-3\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}$ respectively, find the position vector of the point P which divides $\mathbf{AB}$ externally in the ratio $5:3$. **(06 marks)**

b) Find the Cartesian equation of a plane through the origin

parallel to the lines $\mathbf{r} = \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 4 \\ -5 \\ -8 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 7 \\ -6 \end{pmatrix}$.

(06 marks)

13 a) Find the square root of $5 + 12i$. **(06 marks)**

b) Given that $z_1 = 6\left(\cos\frac{5}{12}\pi + i\sin\frac{5}{12}\pi\right)$ and $z_2 = 3\left(\cos\frac{1}{4}\pi + i\sin\frac{1}{4}\pi\right)$,

find $z_1 z_2$ and $\frac{z_1}{z_2}$ in the form $x + yi$. **(06 marks)**

14 a) Evaluate: $\int_0^{\frac{\pi}{2}} \sin 7x \cos 5x \, dx$ **(05 marks)**

b) Use the substitution $x = 2\sin\theta$ to show that $\int_0^1 \frac{x^2}{\sqrt{4-x^2}} dx = \frac{\pi}{3} - \frac{\sqrt{3}}{2}$.

(07 marks)

15. Sketch the curve $y = \frac{4 + 3x - x^2}{x - 8}$, clearly find the nature of the turning points and state their asymptotes. ***(12 marks)***

16. a) Given that $y = 0$ when $x = 1$, solve the differential equation $xy\frac{dy}{dx} = y^2 + 4$ obtaining an expression for y^2 in terms of x.

(05 marks)

b) Solve the differential equation $\frac{dy}{dx} + \frac{xy}{1+x^2} = x$, given that $y = 1$ when $x = 0$. ***(07 marks)***

END